Math 139
Burger

Midterm 1 Solutions

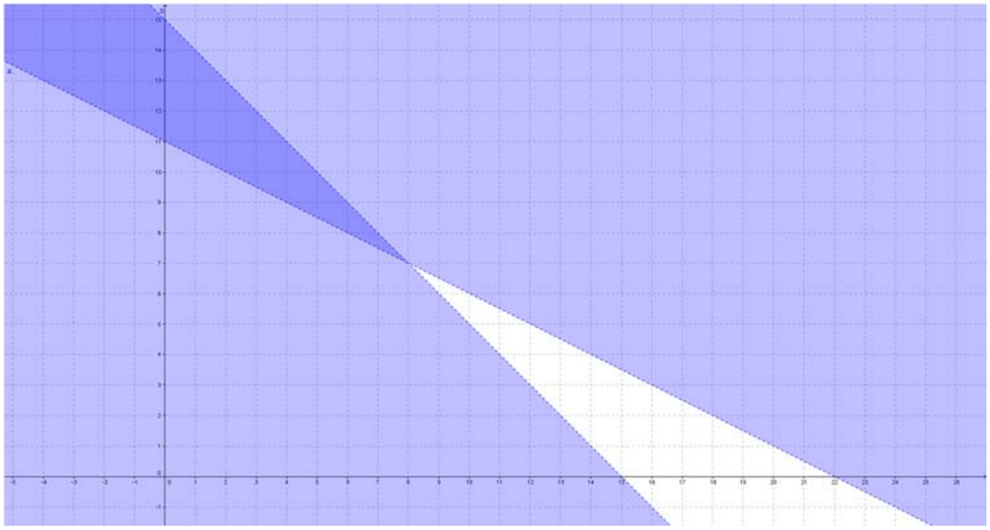
The class average score was $\bar{x} \approx 83.4$ with a standard deviation of $\sigma \approx 10.4$, which means roughly 70% of the class scored between 73 and 94 on the exam. Not bad!

(20pts) <u>Solve by graphing a system of inequalities.</u> (Guess-and-check will not count)

1. JoAnn likes her job as a baby-sitter, but it pays only \$3 per hour. She has been offered a job as a tutor that pays \$6 per hour. Because of school, her parents only allow her to work **less than** 15 hours per week.
   a. What is the <u>most profitable</u> combination of hours JoAnn can tutor and baby-sit and earn **more than** \$66 per week? **(whole # hours only)**

$$3x + 6y > 66 \to x + 2y > 22 \to y > \frac{-1}{2}x + 11$$

$$x + y < 15 \to y < -x + 15$$



The solution (0,14) is the best answer, according to how I worded things, 'whole # hours,' although I meant to word it as 'natural # hours,' which would give the most maximal solution as (1,13).

   b. How much does she make?

$$(0{,}14) \to 6(14) = \$84$$

And $(1{,}13) \to 3(1) + 6(13) = 3 + 73 = \$81$

Both answers were acceptable.

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(20pts)

2. Find the imaginary parts of two of the complex solutions to the following cubic equation (Hint: consider the rational roots theorem to get started since one of the roots must be rational).

$$2x^3 - 3x^2 + 4x + 3 = 0$$

By the rational roots theorem, the possible rational roots are:

$$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$$

after checking, it is found that $-\frac{1}{2}$ is a root since,

$$2\left(\frac{-1}{2}\right)^3 - 3\left(\frac{-1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 3 = -\frac{1}{4} - \frac{3}{4} - 2 + 3 = -1 + 1 = 0$$

So, by the factor theorem, we know that $(x + \frac{1}{2})$ must divide $2x^3 - 3x^2 + 4x + 3$;

but since the leading coefficient is 2, then we use $2 \cdot \left(x + \frac{1}{2}\right) = (2x + 1)$ as the factor dividing $2x^3 - 3x^2 + 4x + 3$.

After doing base-x, we find that $2x^3 - 3x^2 + 4x + 3 = (2x + 1) \cdot (x^2 - 2x + 3)$.

$x^2 - 2x + 3$ is not factorable into integer coefficients, so using the quadratic formula:

$$x = \frac{-(-2) \pm \sqrt{-8}}{2} = 1 \pm i\sqrt{2}$$

Imaginary parts: $\{ \quad \sqrt{2} \quad , \quad -\sqrt{2} \quad \}$

Some of you still forget that $Im(a + ib) = b$ not bi.

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(20 pts)

3. Solve $21x^{2/5} + 23x^{1/5} = 20$. Give solutions in decimal-form rounded to two places (5pt deduction for failure to express answers as decimals!).

Letting $m = x^{1/5} \rightarrow 21m^2 + 23m - 20 = 0$ which factors, using the cloud, as:

$(3m + 5)(7m - 4) = 0$, although most of you used the quadratic formula.

Using the zero product principle, this means that $-\frac{5}{3}$ and $\frac{4}{7}$ are the solutions for m.

But, $m = x^{1/5} \rightarrow -\frac{5}{3} = x^{1/5}$ and $\frac{4}{7} = x^{1/5}$. To solve for x we raise each side of the equations to the 5th power.

$$x = \left(-\frac{5}{3}\right)^5 \approx (-1.67)^5 = -12.99$$

and

$$x = \left(\frac{4}{7}\right)^5 \approx (0.57)^5 = 0.06$$

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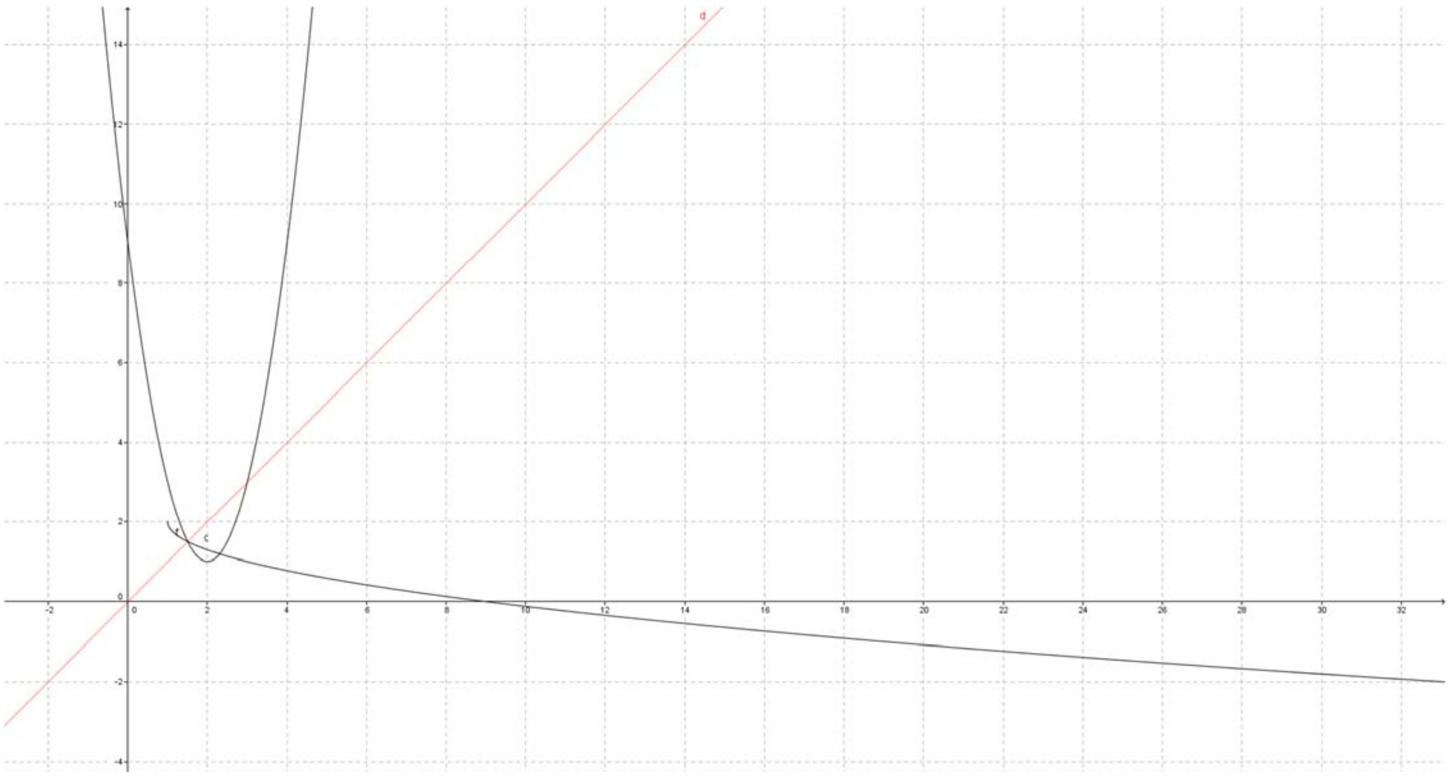
(20pts)

4. a) Put the function $f(x) = 2x^2 - 8x + 9$ into standard form $y = a(x-h)^2 + k$ and sketch ONLY the part of its graph having domain $(-\infty, 2]$.

$$2x^2 - 8x + 9 = 2(x^2 - 4x + 4) - 8 + 9 = 2(x-2)^2 + 1$$

hence the vertex is $(2,1)$.



I couldn't figure out how to restrict the domain in geogebra, but I meant for you to erase the right half of the vertical parabola. As you see, the inverse function, which will exist now, is the horizontal branch shown and having formula:

$$f^{-1}(x) = 2 - \sqrt{\frac{x-1}{2}}$$

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(20pts)

5. Completely factor the following polynomial as much as possible while still having integer coefficients, and sketch its graph with roots and y-intercept labeled. (Hint: consider the rational roots theorem to get started).

$$f(x) = 3x^3 + 2x^2 - 7x + 2$$

Using the rational roots theorem, it is seen that 1 is a root, since

$$3 + 2 - 7 + 2 = 5 - 7 + 2 = -2 + 2 = 0$$

Now, by the factor theorem, we know that $(x - 1)$ divides $f(x)$, so using base-x, we find that:

$3x^3 + 2x^2 - 7x + 2 = (x - 1) \cdot (3x^2 + 5x - 2)$
which factors more, using the cloud, as $(x - 1) \cdot (3x - 1)(x + 2) = f(x)$, and we

then notice we have the roots,

1, $\frac{1}{3}$ and -2, and so we are ready to graph: